

Cosmological Signatures of the Interaction between Dark-Energy and Massive Neutrinos

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We investigate whether interaction between massive neutrinos and quintessence scalar field is the origin of the late time accelerated expansion of the universe. We present cosmological perturbation theory in neutrinos probe interacting dark-energy models, and calculate cosmic microwave background anisotropies and matter power spectrum. In these models, the evolution of the mass of neutrinos is determined by the quintessence scalar field, which is responsible for the cosmic acceleration today. We consider several types of scalar field potentials and put constraints on the coupling parameter between neutrinos and dark energy. Assuming the flatness of the universe, the constraint we can derive from the current observation is $\sum m_\nu < 0.87\text{eV}$ at the 95 % confidence level for the sum over three species of neutrinos. We also discuss on the stability issue of the our model and on the impact of the scattering term in Boltzmann equation from the mass-varying neutrinos.

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Interoduction: After SNIa[1] and WMAP[2] observations during last decade, the discovery of the accelerated expansion of the universe is a major challenge of particle physics and cosmology. In order to understand unknown 76% components of the critical density of the universe with a negative pressure (dark-energy), the positive cosmological constant term seems to be a serious candidate for the dark energy. In this case the cosmological constant Λ and it's energy density $\Lambda/8\pi G$ remain constant with time and corresponding mass density $\rho_\Lambda = 6.44 \times 10^{-30}(\Omega_\Lambda/0.7)(h/0.7)\text{gcm}^{-3}$, where h is the Hubble constant H_0 expressed in units of $100\text{ km s}^{-1}\text{Mpc}^{-1}$ and $\Omega_\Lambda = 0.76$. Although cold dark matter model with a positive cosmological constant [3] (ΛCDM) provides an excellent explanation of the SN1a data, the present value of Λ is 10^{123} times smaller than the value predicted by the particle physics model. Among many alternative candidates for dark energy[4, 5, 6] the scalar field model like quintessence is a simple model with time dependent w , which is generally larger than -1 . Because the different w leads to a different expansion history of the universe, the geometrical measurements of cosmic expansion through observations of SNIa, CMB, and Baryon Acoustic Oscillations (BAO) can give us tight constraints on w . Further, if the dark energy is dynamical component like a scalar field, it should carry its density fluctuations. Thus, the probes of density fluctuations near the present epoch, such as cross correlation studies of the integrated Sachs-Wolfe effect [7, 8] and the power of Large Scale Structure (LSS) [9], can also provide useful information to discriminate between cosmological constant and others. Yet, current observational data can give only poor

constraints on the properties of dark energy fluctuations [10, 11]. Another interesting way to study the scalar field dark energy models is to investigate the coupling between the dark energy and the other matter fields. In fact, a number of models which realize the interaction between dark energy and dark matter, or even visible matters, have been proposed so far [12, 13, 14, 15, 16]. Observations of the effects of these interactions will offer an unique opportunity to detect a cosmological scalar field [12, 17].

Interacting Dark-Energy with Neutrinos: In this letter we investigate the cosmological implication of an idea of the dark-energy interacting with neutrinos [18, 19]. For simplicity, we consider the case that dark-energy and neutrinos are coupled such that the mass of the neutrinos is a function of the scalar field which drives the late time accelerated expansion of the universe. In previous works [18, 19], potential term was treated as a dynamical cosmology constant, which can be applicable for the dynamics near present epoch, but kinetic energy term has been ignored in thier discussions. However the kinetic contributions become important to describe cosmological perturbations in early stage of universe, which is fully considered in our work.

Equations for quintessence scalar field are given by

$$\ddot{\phi} + 2\mathcal{H}\dot{\phi} + a^2 \frac{dV_{\text{eff}}(\phi)}{d\phi} = 0, \quad (1)$$

$$V_{\text{eff}}(\phi) = V(\phi) + V_1(\phi), \quad (2)$$

$$V_1(\phi) = a^{-4} \int \frac{d^3q}{(2\pi)^3} \sqrt{q^2 + a^2 m_\nu^2(\phi)} f(q), \quad (3)$$

$$m_\nu(\phi) = \bar{m}_i e^{\beta \frac{\phi}{M_{\text{Pl}}}}, \quad (4)$$

where $V(\phi)$ is the potential of quintessence scalar field, $V_1(\phi)$ is additional potential due to the coupling to neutrino particles [19, 20], and $m_\nu(\phi)$ is the mass of neutrino coupled to the scalar field. \mathcal{H} is \dot{a}/a , where the dot represents

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resents the derivative with respect to the conformal time τ .

Here we consider three different types of the quintessence potential: (1) inverse power law potentials (Model I), (2) SUGRA type potential models (Model II), (3) exponential type potentials (Model III), which are given, respectively:

$$M^4 \left(\frac{M_{pl}}{\phi} \right)^\alpha ; M^4 \left(\frac{M_{pl}}{\phi} \right)^\alpha e^{3\phi^2/2M_{pl}^2} ; M^4 e^{-\alpha(\frac{\phi}{M_{pl}})}. \quad (5)$$

Energy densities of mass varying neutrino (MVN) and quintessence scalar field are described as

$$\rho_\nu = a^{-4} \int \frac{d^3 q}{(2\pi)^3} \sqrt{q^2 + a^2 m_\nu^2} f_0(q), \quad (6)$$

$$3P_\nu = a^{-4} \int \frac{d^3 q}{(2\pi)^3} \frac{q^2}{\sqrt{q^2 + a^2 m_\nu^2}} f_0(q), \quad (7)$$

$$\rho_\phi = \frac{1}{2a^2} \dot{\phi}^2 + V(\phi), \quad P_\phi = \frac{1}{2a^2} \dot{\phi}^2 - V(\phi). \quad (8)$$

From equations (6) and (7), the equation of motion for the background energy density of neutrinos is given by

$$\dot{\rho}_\nu + 3\mathcal{H}(\rho_\nu + P_\nu) = \frac{\partial \ln m_\nu}{\partial \phi} \dot{\phi}(\rho_\nu - 3P_\nu). \quad (9)$$

The evolution of neutrinos requires solving the Boltzmann equations in the case: [21, 22]:

$$\frac{dq}{d\tau} = -\frac{1}{2} h_{ij} q n^i n^j - a^2 \frac{m}{q} \frac{\partial m}{\partial x^i} \frac{dx^i}{d\tau}. \quad (10)$$

Our analytic formula in eq.(10) is different from those of [32] and [33], since they have omitted the contribution of the varying neutrino mass term. The first order Boltzmann equations written in the synchronous gauge reads [23]:

$$\begin{aligned} \frac{\partial \Psi}{\partial \tau} + i \frac{q}{\epsilon} (\hat{\mathbf{n}} \cdot \mathbf{k}) \Psi + \left(\dot{\eta} - (\hat{\mathbf{k}} \cdot \hat{\mathbf{n}})^2 \frac{\dot{h} + 6\dot{\eta}}{2} \right) \frac{\partial \ln f_0}{\partial \ln q} \\ = -i \frac{q}{\epsilon} (\hat{\mathbf{n}} \cdot \mathbf{k}) k \delta \phi \frac{a^2 m^2}{q^2} \frac{\partial \ln m}{\partial \phi} \frac{\partial \ln f_0}{\partial \ln q}. \end{aligned} \quad (11)$$

The Boltzmann hierarchy for neutrinos, obtained expanding the perturbation Ψ in a Legendre series can be written as [21, 22] :

$$\dot{\Psi}_0 = -\frac{q}{\epsilon} k \Psi_1 + \frac{\dot{h}}{6} \frac{\partial \ln f_0}{\partial \ln q}, \quad (12)$$

$$\dot{\Psi}_1 = \frac{1}{3} \frac{q}{\epsilon} k (\Psi_0 - 2\Psi_2) + \kappa, \quad (13)$$

$$\dot{\Psi}_2 = \frac{1}{5} \frac{q}{\epsilon} k (2\Psi_1 - 3\Psi_3) - \left(\frac{1}{15} \dot{h} + \frac{2}{5} \dot{\eta} \right) \frac{\partial \ln f_0}{\partial \ln q}, \quad (14)$$

$$\dot{\Psi}_\ell = \frac{q}{\epsilon} k \left(\frac{\ell}{2\ell+1} \Psi_{\ell-1} - \frac{\ell+1}{2\ell+1} \Psi_{\ell+1} \right). \quad (15)$$

where

$$\kappa = -\frac{1}{3} \frac{q}{\epsilon} k \frac{a^2 m^2}{q^2} \delta \phi \frac{\partial \ln m_\nu}{\partial \phi} \frac{\partial \ln f_0}{\partial \ln q}. \quad (16)$$

Constraints on the MaVaNs parameters: As was discussed in the introduction, the coupling between cosmological neutrinos and dark energy quintessence could modify the CMB and matter power spectra significantly. It is therefore possible and also important to put constraints on coupling parameters from current observations. For this purpose, we use the WMAP3 [24, 25] and 2dFGRS [26] data sets.

The flux power spectrum of the Lyman- α forest can be used to measure the matter power spectrum at small scales around $z \lesssim 3$ [27, 28]. It has been shown, however, that the resultant constraint on neutrino mass can vary significantly from $\sum m_\nu < 0.2\text{eV}$ to 0.4eV depending on the specific Lyman- α analysis used [29]. The complication arises because the result suffers from the systematic uncertainty regarding to the model for the intergalactic physical effects, i.e., damping wings, ionizing radiation fluctuations, galactic winds, and so on [30]. Therefore, we conservatively omit the Lyman- α forest data from our analysis.

Because there are many other cosmological parameters than the MaVaNu parameters, we follow the Markov Chain Monte Carlo (MCMC) global fit approach [31] to explore the likelihood space and marginalize over the nuisance parameters to obtain the constraint on parameter(s) we are interested in. Our parameter space consists of

$$\vec{P} \equiv (\Omega_b h^2, \Omega_c h^2, H, \tau, A_s, n_s, m_i, \alpha, \beta), \quad (17)$$

where $\omega_b h^2$ and $\Omega_c h^2$ are the baryon and CDM densities in units of critical density, H is the hubble parameter, τ is the optical depth of Compton scattering to the last scattering surface, A_s and n_s are the amplitude and spectral index of primordial density fluctuations, and (m_i, α, β) are the parameters of MaVaNs.

TABLE I: Global analysis data within 1σ deviation for different types of the quintessence potential.

Quantities	Model I	Model II	Model III	WMAP-3 data
$\Omega_B h^2 [10^2]$	2.21 ± 0.07	2.22 ± 0.07	2.21 ± 0.07	2.23 ± 0.07
$\Omega_{CDM} h^2 [10^2]$	11.10 ± 0.62	11.10 ± 0.65	11.10 ± 0.63	12.8 ± 0.8
H_0	65.97 ± 3.61	65.37 ± 3.41	65.61 ± 3.26	72 ± 8
Z_{re}	10.87 ± 2.58	10.89 ± 2.62	11.07 ± 2.44	—
α	< 2.63	< 7.78	< 0.92	—
β	< 0.46	< 0.47	< 0.58	—
n_s	0.95 ± 0.02	0.95 ± 0.02	0.95 ± 0.02	0.958 ± 0.016
$A_s [10^{10}]$	20.66 ± 1.31	20.69 ± 1.32	20.72 ± 1.24	—
$\Omega_Q [10^2]$	68.54 ± 4.81	67.90 ± 4.47	68.22 ± 4.17	71.6 ± 5.5
$Age/Gyrs$	13.95 ± 0.20	13.97 ± 0.19	13.69 ± 0.19	13.73 ± 0.16
$\Omega_{MVN} h^2 [10^2]$	< 0.44	< 0.48	< 0.48	$< 1.97 (95\% C.L.)$
τ	0.08 ± 0.03	0.08 ± 0.03	0.09 ± 0.03	0.089 ± 0.030

Larger β will generally lead larger m_ν in the early universe. This means that the effect of neutrinos on the den-

sity fluctuation of matter becomes larger leading to the larger damping of the power at small scales. A complication arise because the mass of neutrinos at the transition from the ultra-relativistic regime to the non-relativistic one is not a monotonic function of β . Even so, the coupled neutrinos give larger decrement of small scale power, and therefore one can limit the coupling parameter from the large scale structure data.

As shown in table I, we find no observational signature which favors the coupling between MaVaNs and quintessence scalar field, but we don't need to finetune

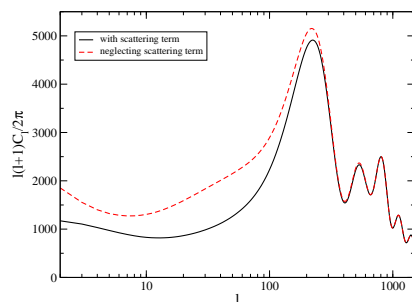


FIG. 1: Differences between the CMB power spectra with and without the scattering term in the geodesic equation of neutrinos with the same cosmological parameters.

Results and discussions: Here we discuss two important points of this work and results of our analysis : (a) the impact of the scattering term of the Boltzmann Equation, (b) the instability issue in our models, and (c) neutrino mass bounds in the interacting neutrino dark-energy models.

(a) Impact of the new scattering term: Recently, perturbation equations for the MaVaNs models were nicely presented by Brookfield et al. [32], (see also [33]) which are necessary to compute CMB and LSS spectra. A main difference here from their works is that we correctly take into account the scattering term in the geodesic equation of neutrinos, which was omitted there (see, however, [34]). Because the term is proportional to $\frac{\partial m}{\partial x}$ and first order quantity in perturbation, our results and those of earlier works [32, 33] remain the same in the background evolutions. However, as will be shown in the appendix, neglecting this term violates the energy momentum conservation law at linear level leading to the anomalously large ISW effect. Because the term becomes important when neutrinos become massive, the late time ISW is mainly affected through the interaction between dark energy and neutrinos. Consequently, the differences show up at large angular scales. In Fig. (1), the differences are shown with and without the scattering term. The early ISW can also be affected by this term to some

the coupling parameters, and obtain the upper limit on the coupling parameter β as

$$\beta < 1.11, 1.36, 1.53 \quad (2\sigma), \quad (18)$$

and the present mass of neutrinos is also limited to

$$\Omega_\nu h_{\text{today}}^2 < 0.0095, 0.0090, 0.0084 \quad (2\sigma), \quad (19)$$

for models I, II and III, respectively.

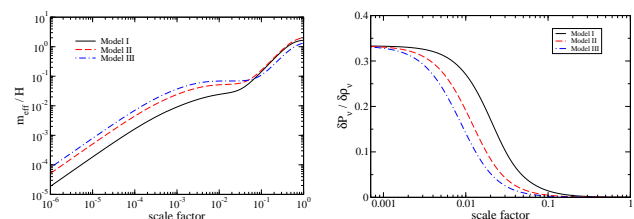


FIG. 2: (Left panel): Typical evolution of the effective mass of the quintessence scalar field relative to the Hubble scale, for all models considered in this paper. (Right panel): Typical evolution of the sound speed of neutrinos $c_s^2 = \delta P_\nu / \delta \rho_\nu$ with the wavenumber $k = 2.3 \times 10^{-3} \text{ Mpc}^{-1}$, for models as indicated. The values stay positive stating from 1/3 (relativistic) and neutrinos are stable against the density fluctuation.

extent in some massive neutrino models and the height of the first acoustic peak could be changed. However, the position of the peaks stays almost unchanged because the background expansion histories are the same.

(b) Instability issue: As shown in [35, 36], some class of models with mass varying neutrinos suffers from the adiabatic instability at the first order perturbation level. This is caused by an additional force on neutrinos mediated by the quintessence scalar field and occurs when its effective mass is much larger than the Hubble horizon scale, where the effective mass is defined by $m_{\text{eff}}^2 = \frac{d^2 V_{\text{eff}}}{d\phi^2}$. To remedy this situation one should consider an appropriate quintessential potential which has a mass comparable the horizon scale at present, and the models considered in this paper are the case [32]. Interestingly, some authors have found that one can construct viable MaVaNs models by choosing certain couplings and/or quintessential potentials [37, 38, 39]. Some of these models even realises $m_{\text{eff}} \gg H$. In Fig.(2), masses of the scalar field relative to the horizon scale m_{eff}/H are plotted. We find that $m_{\text{eff}} < H$ for almost all period and the models are stable. We also depict in Fig.(2) the sound speed of neutrinos defined by $c_s^2 = \delta P_\nu / \delta \rho_\nu$ with a wavenumber $k = 2.3 \times 10^{-3} \text{ Mpc}^{-1}$.

(c) Neutrino Mass Bounds: When we apply the relation between the total sum of the neutrino masses M_ν and their contributions to the energy density of the uni-

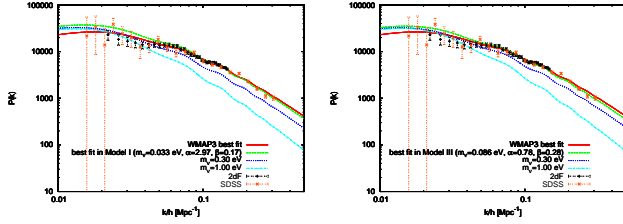


FIG. 3: Examples of the total mass contributions in the matter power spectrum in Model I (Left panel) and Model III (Right panel). For both panels we plot the best fitting lines (green dashed), lines with larger neutrino masses $M_\nu = 0.3$ eV (blue dotted) and $M_\nu = 1.0$ eV (cyan dot-dashed) with the other parameters fixed to the best fitting values. Note that while lines with $M_\nu = 0.3$ eV can fit to the data well by arranging the other cosmological parameters, lines with $M_\nu = 1.0$ eV can not.

verse: $\Omega_\nu h^2 = M_\nu / (93.14 \text{ eV})$, we obtain the constraint on the total neutrino mass: $M_\nu < 0.87 \text{ eV}$ (95% C.L.) in the neutrino probe dark-energy model. The total neutrino mass contributions in the power spectrum is shown in Fig 3, where we can see the significant deviation from observation data in the case of large neutrino masses.

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